MEMOIRS

OF THE

COLLEGE OF SCIENCE

UNIVERSITY OF KYOTO

SERIES A

VOL. XXV, NO. 1

MARCH 1947

KYOTO

1127 m

MEMOIRS OF THE COLLEGE OF SCIENCE, UNIVERSITY OF KYOTO, SERIES, A
Vol. XXV, No. 1, Article 1, 1947.

On the Structure of Inter-crystalline Boundaries of Metals. I

By

Masaharu Kabata

(Received August 10, 1945)

Abstract

The liquid layers at the inter-crystalline boundary and on the outer surface of test piece were confirmed with various metals to be present, by means of the X-ray diffraction method, at least, at the temperatures lower by less than 10°~20°C below the melting point; and the grooving of the inter-crystalline boundary at such a high temperature was ascribed to the sucking up of the inter-crystalline liquid layer to the outer surface of the test piece by the action of the surface tension of the liquid. The presence of the inter-crystalline liquid layer was detected also with bismuth differently. When a test plece of bismuth is cooled from such a high temperature, the expansion due to the crystallization of the inter-crystalline liquid layer is revealed by an uneven bulging of both sides of the inter-crystalline groove as well as by the eruption as small spheres of the interior part of the liquid layer remaining still uncrystallized. According to rough measurements, the thickness of the inter-crystalline liquid layer decreases from several tenth mm. at extreme vicinity of the melting point to almost nil at the temperature 10°~20°C below.

By recrystallization of metals at high temperature a smoothing of the surface of the test piece was observed, and it was ascribed to the sweeping of small surface irregularities by the passage of the intersection of the inter-crystalline liquid layer with the liquaid layer on the outer surface of the test piece.

§ I. *Introduction.* The presence of an amorphous layer on well polished metal surface was pointed out first by Beilby,[1] as is known as Beilby's amorphous hypothesis. Rosenhain[2] and others extended Beilby's hypothesis of amorphous layer to inter-crystalline boundaries of metals, and by assuming the presence of cementing amorphous layer in inter-crystalline boundaries, they explained the fact that the inter-crystalline boundaries are mechanically stronger at room temperature than interior of the crystals with which a metal block is composed.

1. G. Beilby: *Aggregation and Flow of Solids*, (1921).
2. W. Rosenhain and D. Ewen: *J. Inst. Metals*, 8, 149, (1912).

was so suspended with a wire at the lower end of the test piece that the weight situated outside the porcelain tube. To the lower open end of the tube is connected a glass tube filled with water at its bottom. The purpose of this device is to cool suddenly the part of the test piece which has fallen out of the electric furnace being torn at a crystal boundary by the suspended load at high temperature. The rapidity of heating the test piece in the furnace was 5°C per minute from the room temperature to about 20°C below the melting point, and was about 0.5°C per minute above. For taking the diffraction photograph, X-ray tube provided with tungsten target was operated with 80 kV, and a cylindrical camera was employed.

(b) *Experiments*: As the purpose of the X-ray diffraction experiment is to examine the thin layer of the outer flat surface of the test piece and of the sectional surface of inter-crystalline boundary which is developed by tearing, it was carried out with the test pieces (SR), (BI), and (LI) whose density is high. When a test piece consisting of large crystals, whose boundary runs across the entire section of the test piece, is hung in the furnace, in the manner as is stated before and is heated up to a certain high temperature, the test piece get asunder into two pieces at the crystal boundary by being pulled with the load applied to the lower end of the test piece. The piece which has dropped into the water under it cools very rapidly so far as about 15°C, and the other piece remaining in the furnace is made to cool very gradually in it by cutting the electric current flowing through the furnace. By this experiment the temperature at which the tearing of the test piece happens, can be controlled at least to some extent by changing the load applied to the test piece. However below a certain temperature, which is proper to the kind of the metal, the separation of the test piece at the crystal boundary does never take place even with very heavy load.

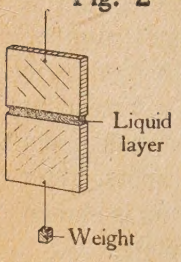
(c) *Results*: The tearing of the test pieces of (SR), (BI) and (LI) at the crystal boundary was done at various temperatures in the manner stated former section. The sectional surfaces of the crystal boundaries of the test pieces which were developed by tearing and their outer flat surface, with both rapidly and slowly cooled ones, were examined by the X-ray reflection method; and entirely similar photographs were obtained with every metal. The

photographs obtained with lead are shown, as examples, in Plate I. As is seen in Fig. 3 and 4 in the plate, slowly cooled specimens gave only ordinary Laue-spots consisting of a small number of intense spots due to a single piece of crystal, with both the outer flat surface and the sectional surface developed by tearing at the crystal boundary. Differing from this, a lot of weak spots appeared, in addition to a small number of intense spots, with both kinds of the surfaces of rapidly cooled specimens, as is shown by the photographs a, b, c, d in Fig. 1 and 2 in the plate.

As to the origin of these weak Laue-spots it is reasonably considered that the surfaces of both kinds of the specimens become covered with a thin layer of the melt at the temperature of tearing and that this molten thin layer solidifies in polycrystalline structure by rapid cooling, having not enough time to be occupied by the base crystal by its growth. It may be differently imagined that the weak Laue-spots in question originate in the small crystals formed by the destruction of the original large crystal by being cooled rapidly. However, as will be seen from the photographs in Plate I, the appearence of the weak Laue-spots is limited to the melting point, and when the heating temperature is lower, we get only a small number of the intense Laue-spots arisen from the original large crystal. This indicates the inadequacy of the latter view. The weak Laue-spots weaken as the heating temperature lowers below the melting point, and become almost undetected at the temperature of about 10°C below the melting point, being more or less different according to the kind of metals. This arises from the thinning of the liquid layer in the inter-crystalline boundaries and on the outer flat surface of the specimen with lowering of the heating temperature; and this point will be more clearly shown later in § IV, where the results of measurements of the thickness of the liquid layer are given.

The liquidification of the outer flat surface of the specimen at high temperature can also be detected in different way. When a plate consisting of only a few number of crystals is suspended vertically in the furnace and is heated up to the temperature of 2° or 3°C below the melting point, there appears a vestige of the flow of the melt on the surface of the specimen, and a bulge at the lower end of the plate which is evidently due to the accumulation of the flowed melt. The photograph, taken in end-on direc-

Fig. 2



tion, of the bulge formed in such a way at lower end of an aluminium plate is shown in Fig. 4, Plate II. As the plate preserved its general form in this case, the fluidification should be condidered to be limited to the outer surface of the plate and the inter-crystalline boundaries, leaving the interior unliquidified. As to the question why the outer surface of the metal is easier to melt than the interior, the writer's opinion is as follow:

If we consider that the metal surface is covered with oxide film, then the boundary between this film and the crystal beneath it will be essentially the same nature as the inter-crystalline boundary; and consequently the melting is as easier at the former boundary as at the latter.

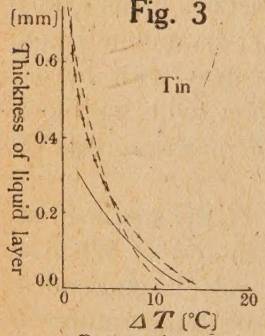
§IV. *The Thickness of the Liquid Layer Existing at the Inter-crystalline Boundary.*

Table II

Metals	Surface tension in dyne/cm	Remark
Sn	612.4	at 226°C in CO
Al	520.0	at 750°C
Bi	465.0	at 265°C in CO

As it has become almost evident from the experiments described above, that with metals the liquid layer exists at the inter-crystalline boundary at the temperatures which are at least a little below the melting point, the writer tried to measure its thickness at various temperatures.

Fig. 3

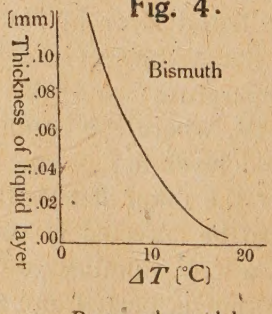


---- Rectangular rod by recrystallization

-·-·- Plate by recrystallization

—— Rectangular rod by casting

Fig. 4.



—— Rectangular rod by casting

The crystal boundary is of course pretty irregular. However the writer selected such specimens from the test pieces in the forms of plate or rectangular rod consisting of large crystals that the crystal boundaries devoid of much irregularity ran almost perpendicularly to the longitudinal direction of the test piece.

If we approximately assume that the crystal boundary is a horizontal plane perpendicular to the longitudinal direction of the test piece and that the liquid layer at the inter-crystalline boundary is of a uniform thickness as is shown by Fig. 2; then the force due to the surface tension of the liquid layer, which is equal to the tearing weight at the moment of tearing, can be calculated very easily. If we represent by γ the surface tension of the liquid, by t the thickness of the liquid layer, by A the area of the horizontal liquid layer which is taken to be the same as the horizontal cross sectional area of the test piece, M the tearing weight including the mass of the piece of the test piece which has fallen by the tearing, and by g the gravity acceleration, then we get $2\gamma A/t = Mg = 980\,M$.

Thus from the values of γ, A and M, the thickness of the liquid layer t can be obtained. The values of surface tension for Sn, Bi and Al at temperatures somewhat higher than the respective melting points, which are given in Landolt-Börnstein's *Physikalisch-Chemische Tabelle*, are tabulated in Table II. The present experiments were carried out in air and at temperatures somewhat lower than the respective melting points, and consequently the values of the surface tension would have been different from those given in the table. However the writer used the values given in the table, by regarding them to be the same order of magnitude, in both cases.

The tearing experiments were made with the test piece (SR), (SI), (AR), (AI) and (BI) by means of the furnace shown in § III, (a). As the tearing temperature is different according to the tearing load, the experiments were made by applying many different loads for a same kind of the test piece. Thus by knowing the tearing load (including the weight of the fallen part of the test piece), the thickness t of the liquid layer was obtained by measuring the cross-sectional area A of the test piece. The relation between the temperature and the thickness of the liquid layer for Sn and Bi is shown in Fig. 3 and 4. In these figures the temperature is represented as its difference from the melting point and is expressed as ΔT.

As is evident from the curves in the figures, the thickness of the liquid layer at the inter-crystalline boundary decreases as the temperature lower from the melting point, and becomes so thin at

the temperature of about 10°~20°C below the melting point that it makes impossible to tear at the crystal boundary. Below this temperature we are unable to detect the presence of the liquid layer by the tearing method or by the X-ray method. However it is inconceivable that the liquid layer disappears abruptly below a certain temperature; and it seems more natural to suppose, as Prof. U. Yoshida has done, that the thin layer at the crystal boundary, be in the state of liquid or of amorphous solid, by far exists when it only becomes thinner as the temperature falls further and further.

It is now almost clarified that the crystal boundary of the melting point. When a test piece of a metal composed of large crystals is heated to such a temperature, the crystal boundary gets grooved as was stated before. Fig. 1, Plate II shows such grooves formed on the surface of an aluminium plate. As will be stated later, this seems to be caused by the fact that the liquid layer formed at the inter-crystalline boundary is sucked up beneath the oxide film on the outer surface of the test piece by virtue of the action of the surface tension.

The writer measured with the test piece of aluminium the depth and the breadth of such grooves at the crystal boundary by means of Leitz's Ultrapak metal microscope, and investigated their relation with the highest temperature to which the test piece is heated up.

The procedure of experiment is as follows: When the temperature of the vertical electric furnace as is shown in § III, (a), reached 630°C, a test piece (AR) was hung into it, and when the temperature attained to the highest temperature required and

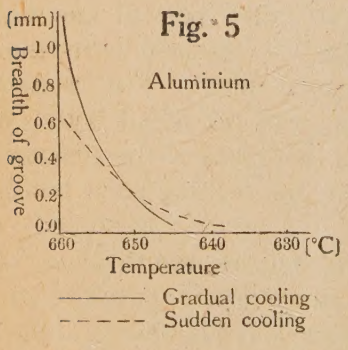


Fig. 5

Aluminium

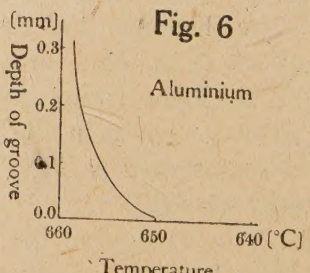


Fig. 6

Aluminium

maintained at that temperature for two minutes, the test piece was either withdrawn up and was left to cool in the air, or cooled suddenly by dropping it into the water under the furnace.

The results of measurements obtained with various heating temperatures are represented by the curves in Fig. 5 and 6. The results of these measurements generally agree well with these obtained by the tearing method; and this shows that the thickness of the liquid layer measured by the tearing method is comparatively reliable. The breadth of the grooves which was measured microscopically by the present method is somewhat larger at high temperatures than the thickness of the liquid layer obtained by the tearing method. This is probably due to the fact that the edges formed by the liquid layer and the outer flat surface of the test piece become rounded and ill defined.

(to be continued)

Plate I *Lead*

Fig. 1 The sudden cooling sectional surface of crystal boundary.

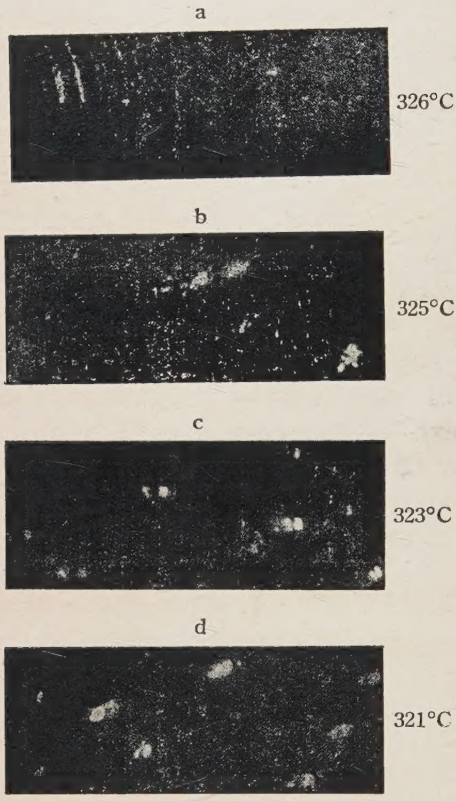


Fig. 2 The outer suaface of sudden cooling specimen.

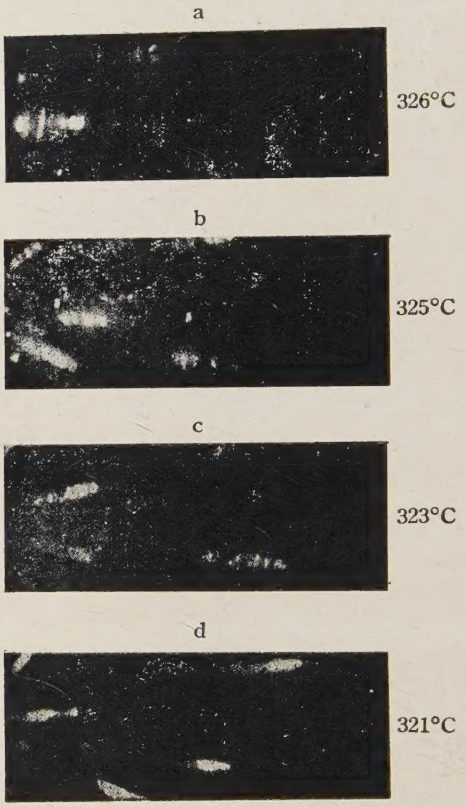


Fig. 3 The slow cooling sectional surface of crystal boundary.

Fig. 4 The outer surface of slow cooling specimen.

Remark : The temperature shows the highest temperature.

MEMOIRS OF THE COLLEGE OF SCIENCE, UNIVERSITY OF KYOTO, SERIES A,
Vol. XXV, No. 1, Article 2, 1947.

One Possibility of Oscillation caused by Electrons in a Constant Magnetic Field

By

Isao Takahashi

(Received November 19, 1946)

Introduction

The author has previously[1] calculated the expressions of dielectric constant and conductivity in the space of a parallel plane condenser, through which electrons flow along a constant magnetic field, and verified the expression of the dielectric constant by comparing it with the data of the experiment in which the author has measured the variation of the dielectric constant with the variation of the magnetic field.

But the detailed discussion of the expression of conductivity has been omitted there.

The consideration of this expression makes the author anticipate the possibility of causing the negative resistance that is, one method of oscillation.

Current between Electrodes

Some of the results which the author has previously[2] made public are quoted here for the present necessity.

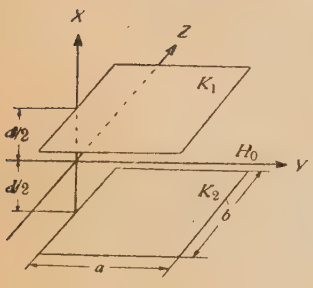
Electrons incident on yz-plane flow between parallel plane electrodes K_1 and K_2 along y-axis, that is, a constant magnetic field of intensity H_0 as in Fig. 1.

Then the current cirulating through K_1, K_2 and the external circuit connected is

1. I. Takahashi: *'Über die Dielektrizitätskonstante des Magneto-aktiven Ionisierten Gases'* (eingegangen am 20ten März, 1941), *These Memoirs*, A, **23**, No. 6, (1942), pp. 373~397.

2. I. Takahashi: *loc. cit.*

Fig. 1



$$J=\int_{t-T}^{t}\frac{nev_x}{d}dt_0-\frac{nes}{d}, \qquad (1)$$

where

t: instant of time,
t_0: instant of incidence,
T: transit time through condenser space,
n: number of electrons incident per unit time,
v_x: velocity component of an electron perpendicular to electrodes,
d: distance between electrodes,
s: displacement of an electron at the exit of the condenser space,
e: charge of an electron,
m: mass of an electron,
c: light velocity,

though the last two do not explicitly appear here.

Thus

$$\frac{J}{ab}=E_0e^{j\omega t}(\sigma+j\omega\Delta\varepsilon), \qquad (2)$$

where $E_0e^{j\omega t}$ is oscillating voltage between K_1 and K_2.

$$\sigma=\frac{ne^2T^2}{mabd}\frac{\omega}{\omega_0}\frac{1}{2}\left\{G(\omega_1T)-G(\omega_2T)\right\}, \qquad (3)$$

$$\Delta\varepsilon=\frac{4\pi ne^2T^2}{mabd}\frac{1}{\omega_0}\frac{1}{2}\left\{F(\omega_1T)+F(\omega_2T)\right\}$$

with

$$\omega_0=\frac{eH_0}{cm}, \qquad (4)$$

$$\left.\begin{aligned}\omega_1&=\omega_0-\omega,\\ \omega_2&=\omega_0+\omega,\end{aligned}\right\} \qquad (5)$$

$$\left.\begin{aligned}G(X)&=\frac{1}{X^2}(1-\cos X),\\ F(X)&=\frac{1}{X}\left(1-\frac{\sin X}{X}\right).\end{aligned}\right\} \qquad (6)$$

This expression of $\Delta\varepsilon$ has been previously verified by compar-

ing it with the data of the experiment made by the author.

Character of σ

The graph of $G(X)$ with respect to X is plotted in Fig. 2.

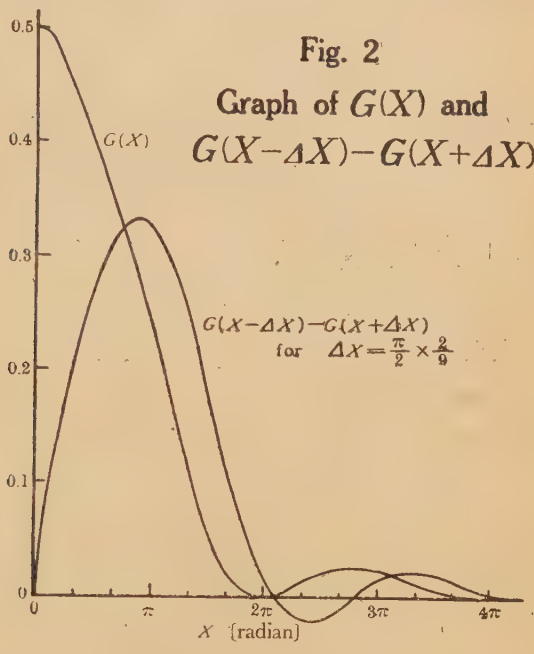


Fig. 2
Graph of $G(X)$ and $G(X-\Delta X)-G(X+\Delta X)$

If we put

$$\left.\begin{aligned}\omega_0 T &\equiv X,\\ \omega T &\equiv \Delta X,\end{aligned}\right\} \qquad (7)$$

then $G(\omega_1 T)\equiv G(X-\Delta X)$ and $G(\omega_2 T)\equiv G(X+\Delta X)$ are displaced to the right and to the left of $G(X)$ by ΔX respectively.

On the other hand, the mininum values of $G(X)$ occurs at $X=2n\pi$ with n: integers, and the maximum values of $G(X)$ at $X=(2n-1)\pi-\theta_n$ with n: integers, where $\theta_n \to 0$ with n becoming larger. Therefore we can see the possibility of $\sigma<0$ rises when the vicinity of the maximum of $G(X+\Delta X)$ overlaps that of the minimum of $G(X-\Delta X)$.

For example, in case

$$X+\Delta X=3\pi-\theta_1 \fallingdotseq 3\pi-\frac{\pi}{2}\times\frac{2}{9},$$

$$X-\Delta X=2\pi,$$

that is

$$\left.\begin{aligned}\Delta X&=\omega T=\tfrac{1}{2}(\pi-\theta_1)\fallingdotseq\tfrac{1}{2}(\pi-\pi/9),\\ X&=\omega_0 T=\tfrac{1}{2}(5\pi-\theta_1)\fallingdotseq\tfrac{1}{2}(5\pi-\pi/9),\end{aligned}\right\}\qquad(8)$$

$|\sigma|$ of $\sigma<0$ becomes the largest.

The graph of $G(X-\Delta X)-G(X+\Delta X)$ with respect to X is plotted in Fig. 2, where $\Delta X=\frac{1}{2}(\pi-\theta_1)$.

For $\omega=2\pi\times 3\times 10^9$, that is, wave length 10cm, the relation (8) become

$$\left.\begin{aligned}T&=\frac{1}{4\pi\times 3\times 10^9}(\pi-\theta_1)\ \text{in sec.},\\ H_0&=\frac{1}{2T}(5\pi-\theta_1)\frac{cm}{e}\ \text{in Gauss.}\end{aligned}\right\}$$

These are realizable figures.

Conclusion

The author proposes one method of producing oscillation which he has not yet experimentally verified in spite of his will.

The author hopes to be given kind instructions by the readers.

MEMOIRS OF THEC OLLEGE OF SCIENCE, UNIVERSITY OF KYOTO, SERIES A,
Vol. XXV, No. 1, Article 3, 1947.

On the Possibility of Barkhausen-Kurz Oscillation. I

By

Isao Takahashi

(Received November 19, 1946)

Introduction

The frequency of Barkhausen-Kurz oscillation is nearly equal to the inverse of the period of the pendulum motion of an electron or to its integral multiples. This is an experimental fact and it can easily be imagined that the frequencies are such ones, when oscillation occurs. But this relation of frequency can never afford any explanation to the occurrence of oscillation, because if the electrons which can contribute to the oscillation are respectively in different phases of motion, their effects are nothing in all. On the other hand, the occurrence of oscillation as an experimental fact must be attributed to the existence of the negative resistance, in an electron tube.

The author has calculated the currents between electrodes in an electron tube for two cases. The first case is one when no electrons are caught by the anode by means of sufficient negative anode voltage and the second is one when the electrons of larger amplitude are caught by the anode. By use of these expressions, the author demonstrates that the negative resistance can occur between electrodes at the expected frequencies. In this paper, only the first case is treated and the second case is to appear in part II.

Currents between Electrodes, when no Electrons are caught by the Anode

We assume the parallel plane electrodes as (a), (b) and (c) in Fig. 1, and neglect the effect of the space charges. In the following, where v_1 and v_2 are assumed to be small against V_g, we cal-

culate, neglecting the terms of the order of $(v/V_g)^2$ and the higher terms.

The notations to be used are summarized:

T_1: transit time from K to G when $v=0$,

T_2: transit time from G to turning point when $v=0$.

$$T_0=2(T_1+T_2),$$

$$\alpha_1=\frac{eV_g}{md_1},\quad \alpha_2=\frac{e(V_g+V_a)}{md_2},$$

$$\beta_1=\frac{ev_1}{md_1},$$

$$\beta_2=\begin{cases}\dfrac{ev_1}{md_2} & \text{for (a),}\\ \dfrac{e(v_1+v_2)}{md_2} & \text{for (b),}\\ \dfrac{-ev_2}{md_2} & \text{for (c),}\end{cases}$$

m, $-e$ being the mass and the charge of an electron.

n: the number of electrons emitted per unit time by K to contribute to the oscillation.

The relation

$$\alpha_1T_1=\alpha_2T_2 \qquad (1)$$

exists.

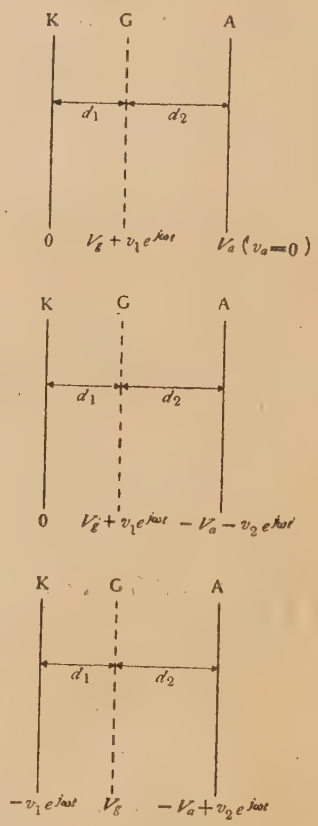
From the equation of motion, we obtain the following velocity expressions for an electron leaving K at the instant $t=t_0$:

from K to G,

$$\begin{aligned}u_1&=u_1'+u_1'',\\ u_1'&=\alpha_1(t-t_0),\\ u_1''&=\frac{\beta_1}{j\omega}(e^{j\omega t}-e^{j\omega t_0});\end{aligned} \qquad (2)$$

Voltage distribution according to the circuit

Fig. 1



A: anode
G: grid
K: cathode
V_g: positive grid voltage
$-V_a$: negative anode voltage
$ve^{j\omega t}$: oscillating voltage
d: distance between electrodes

from G to turning point and thence to G again:

$$\begin{aligned}
&u_2=u_2'+u_2'',\\
&u_2'=\alpha_2(t_0-t)+(\alpha_1+\alpha_2)T_1,\\
&u_2''=(\alpha_1+\alpha_2)\delta T_1+\frac{\beta_1}{j\omega}e^{j\omega t_0}(e^{j\omega T_1}-1),\\
&\delta T_1(t_0)=\frac{\beta_1}{\alpha_1 T_1\omega^2}e^{j\omega t_0}(e^{j\omega T_1}-1-j\omega T_1);
\end{aligned}\tag{3}$$

and from G to K,

$$\begin{aligned}
&u_3=u_3'+u_3'',\quad u_3'=\alpha_1(t-t_0)-\alpha_1 T_0,\\
&u_3''=-(\alpha_1+\alpha_2)\delta T_2+\frac{\beta_1}{j\omega}(e^{j\omega t}-e^{j\omega(t_0+T+2T_2)})\\
&\qquad+\frac{\beta_1}{j\omega}e^{j\omega t_0}(e^{j\omega T_1}-1)-\frac{\beta_2}{j\omega}e^{j\omega(t_0+T_1)}(e^{j2\omega T_2}-1),\\
&\delta T_2(t_0)=\frac{2\alpha_1}{\alpha_2}\delta T_1(t_0)+\frac{2\beta_1}{\alpha_2 j\omega}e^{j\omega t_0}(e^{j\omega T_1}-1)\\
&\qquad+\frac{\beta_2}{\alpha_2 T_2\omega^2}e^{j\omega(t_0+T_1)}(e^{j2\omega T_2}-1-j2\omega T_2).
\end{aligned}\tag{4}$$

$\delta T_1(t_0)$ and $\delta T_2(t_0)$ are such that the electron passes through G at $t=t_0+T_1+\delta T_1(t_0)$ on the way to A and at $t=t_0+T_1+2T_2+\delta T_1(t_0)+\delta T_2(t_0)$ on the way back, when δT_1 and δT_2 mean their real parts here.

If the instant at which the electron comes back to K is $t_0+T_0+\delta T_0(t_0)$, $\delta T_0(t_0)$ is not needed to the calculation of currents, as can be indicated later and so we do not seek its functional form. Now, the current J_A flowing into G from outside on account of electron motions between G and A is

$$J_A=\frac{-ne}{d_2}\int_{t_0''}^{t_0'}u_2 dt_0,$$

where t_0' is the t_0 of the electron which passes through G at t on the way to A and t_0'' the t_0 of the electron which passes through G at t on the way back. Therefore,

$$\begin{aligned}
t_0'&=t-T_1-\delta T_1(t_0')\fallingdotseq t-T_1-\delta T_1',\\
t_0''&=t-T_1-2T_2-\delta T_1(t_0'')-\delta T_2(t_0'')\\
&\fallingdotseq t-T_1-2T_2-\delta T_1''-\delta T_2'',
\end{aligned}$$

with

$$\delta T_1' \equiv \delta T_1(t-T_1), \qquad \delta T_1'' \equiv \delta T_1(t-T_1-2T_2),$$
$$\delta T_2'' \equiv \delta T_2(t-T_1-2T_2). \tag{5}$$

Then

$$J_A = \frac{-ne}{d_2}\left\{\int_{t-T_1-2T_2-(\delta T_1''+\delta T_2'')}^{t-T_1-2T_2} u_2 dt_0 + \int_{t-T_1-2T_2}^{t-T_1} u_2 dt_0 - \int_{t-T_1-\delta T_1'}^{t-T_1} u_2 dt_0\right\}.$$

Let the d.c. part of J_A be J_1, and the oscillating part of frequency ω be i_1, then

$$J_1 = \frac{-ne}{d_2}\int_{t-T_1-2T_2}^{t-T_1} u_2' dt_0 = 0,$$

$$i_1 = \frac{-ne}{d_2}\left\{\int_{t-T_1-2T_2-(\delta T_1''+\delta T_2'')}^{t-T_1-2T_2} u_2' dt_0 - \int_{t-T_1-\delta T_1'}^{t-T_1} u_2' dt_0 + \int_{t-T_1-2T_2}^{t-T_1} u_2'' dt_0\right\},$$

$$\therefore\ i_1 = \frac{ne}{d_2\omega^2}\beta_1 e^{j\omega t}\left\{\frac{\beta_2}{\beta_1}(2-2e^{-j2\omega T_2}-j2\omega T_2-j2\omega T_2 e^{-j2\omega T_2})\right.$$
$$+(1-e^{-j\omega T_1}-j\omega T_1 e^{-j\omega T_1})\left[1+\left(\frac{2\alpha_1+\alpha_2}{\alpha_2}\right)e^{-j2\omega T_2}-\left(\frac{\alpha_1+\alpha_2}{\alpha_2}\right)\right.$$
$$\left.\left.\times\frac{1-e^{-j2\omega T_2}}{j\omega T_1}\right]+(1-e^{-j\omega T_1})(1-e^{-j2\omega T_2}-j2\omega T_2 e^{-j2\omega T_2})\right\}. \tag{6}$$

In the next place, the current J_K flowing into G from outside on account of electron motions between G and K is

$$J_K = \frac{ne}{d_1}\left\{\int_{t-T_1-\delta T_1'}^{t} u_1 dt_0 + \int_{t_0'''}^{t-T_1-2T_2-(\delta T_1''+\delta T_2'')} u_3 dt_0\right\},$$

where t_0''' is the t_0 of the electron which come back to K at t, i.e.

$$t_0''' = t-T_0-\delta T_0(t_0'') \fallingdotseq t-T_0-\delta T_0''', \quad [\delta T_0''' \equiv \delta T_0(t-T_0)].$$

$$J_K = \frac{ne}{d_1}\left\{\int_{t-T_1-\delta T_1'}^{t-T_1} u_1 dt_0 + \int_{t-T_1}^{t} u_1 dt_0 + \int_{t-T_0-\delta T_0'''}^{t-T_0} u_3 dt_0 + \int_{t-T_0}^{t-T_1-2T_2} u_3 dt_0 - \int_{t-T_1-2T_2-(\delta T_1''+\delta T_2'')}^{t-T_1-2T_2} u_3 dt_0\right\},$$

and the d.c. part J_2 of J_K is

$$J_2 = \frac{ne}{d_1}\left\{\int_{t-T_1}^{t} u_1' dt_0 + \int_{t-T_0}^{t-T_1-2T_2} u_3' dt_0\right\} = 0;$$

and the oscillating part i_2 of frequency ω of J_K is

$$i_2=\frac{ne}{d_1}\left\{\int_{t-T_1-\delta T_1'}^{t-T_1}u_1'dt_0+\int_{t-T_0-\delta T_0'''}^{t-T_0}u_3'dt_0-\int_{t-T_1-2T_2-(\delta T_1''+\delta T_2'')}^{t-T_1-2T_2}u_3'dt_0+\int_{t-T_1}^{t}u_1''dt_0+\int_{t-T_0}^{t-T_1-2T_2}u_3''dt_0\right\},$$

$$\therefore\quad i_2=\frac{ne}{d_1\omega^2}\beta_1e^{j\omega t}\left\{\frac{\beta_2}{\beta_1}\left[(1-e^{-j2\omega T_2}-j2\omega T_2e^{-j2\omega T_2})-\left(\frac{a_1+a_2}{a_2}\right)\frac{T_1}{T_2}\right.\right.$$
$$\times\frac{1}{j\omega T_1}(1-e^{-j\omega T_1})(1-e^{-j2\omega T_2}-j2\omega T_2e^{-j2\omega T_2})+(1-e^{-j\omega T_1})$$
$$\left.\times(1-e^{-j2\omega T_2})\right]+(1-e^{-j\omega T_1}-j\omega T_1e^{-j\omega T_1})\left[1+\left(\frac{2a_1+a_2}{a_2}\right)e^{-j2\omega T_2}\right]$$
$$+(1-e^{-j\omega T_1})\left[-j2\omega T_2e^{-j2\omega T_2}-\left(\frac{a_1+a_2}{a_2}\right)\frac{1}{j\omega T_1}2(1-j\omega T_1\right.$$
$$\left.-e^{-j\omega T_1})e^{-j2\omega T_2}+(1-e^{-j2\omega T_2}+e^{-j2\omega T_2}\cdot e^{-j\omega T_1})\right]$$
$$\left.+(1-e^{-j\omega T_1}-2j\omega T_1)\right\}. \qquad (7)$$

Here it is noticed that in the calculation of i_2,

$$\int_{t-T_0-\delta T_0'''}^{t-T_0}u_3'dt_0=0$$

holds in the first order, independent of the form of $\delta T_0(t_0)$.

Interelectrode impedances and some numerical examples.

In order that oscillation may occur in the system of an electron tube and an external circuit, it is needed that the negative resistance between electrodes in an electron tube occurs.

For the case when no electrons are caught by the anode, we define interelectrode impedance z_1 between G and A and z_2 between G and K by the following expressions, the suffix a, b, c specifying the circuit related respectively:

$$\frac{1}{z_{1a}}=\frac{i_{1a}}{v_1e^{j\omega t}}=\frac{ne^2}{md_1d_2\omega^2}F_1(X,\ Y,\ \varepsilon_a),$$
$$\frac{1}{z_{2a}}=\frac{i_{2a}}{v_1e^{j\omega t}}=\frac{ne^2}{md_1^2\omega^2}F_2(X,\ Y,\ \varepsilon_a),$$

$$\frac{1}{z_{1b}}=\frac{i_{1b}}{(v_1+v_2)e^{j\omega t}}=\frac{ne^2}{md_1d_2\omega^2}\left(\frac{1}{1+x}\right)F_1(X,\ Y,\ \varepsilon_b),$$
$$\frac{1}{z_{2b}}=\frac{i_{2b}}{v_1e^{j\omega t}}=\frac{ne^2}{md_1^2\omega^2}F_2(X,\ Y,\ \varepsilon_b),$$

$$\frac{1}{z_{1c}}=\frac{-i_{1c}}{v_2e^{j\omega t}}=\frac{ne^2}{md_1d_2\omega^2}\left(\frac{1}{-\varkappa}\right)F_1(X,\ Y,\ \varepsilon_c),$$

$$\frac{1}{z_{2c}}=\frac{i_{2c}}{v_1e^{j\omega t}}=\frac{ne^2}{md_1^2\omega^2}F_2(X,\ Y,\ \varepsilon_c),$$

where $X=\omega T_1,\quad Y=\omega T_2,\quad \varkappa=v_2/v_1,$

$$\varepsilon_a=\frac{\beta_2}{\beta_1}=\frac{d_1}{d_2},\qquad \varepsilon_b=\frac{\beta_2}{\beta_1}=\frac{d_1}{d_2}\frac{v_1+v_2}{v_1}=(1+\varkappa)\frac{d_1}{d_2},$$

$$\varepsilon_c=\frac{\beta_2}{\beta_1}=-\frac{d_1}{d_2}\frac{v_2}{v_1}=-\varkappa\frac{d_1}{d_2},$$

$$F_1(X,\ Y,\ \varepsilon)\equiv\varepsilon(2-2e^{-j2Y}-j2Y-j2Ye^{-j2Y})+(1-e^{-jX}-jXe^{-jX})\times\left[1+\frac{X+2Y}{X}e^{-j2Y}+j\frac{X+Y}{XY}(1-e^{-j2Y})\right]+(1-e^{-jX})(1-e^{-j2Y}-j2Ye^{-j2Y}),\tag{8}$$

$$F_2(X,\ Y,\ \varepsilon)\equiv\varepsilon\Big[(1-e^{-j2Y}-j2Ye^{-j2Y})+j\frac{X+Y}{XY}(1-e^{-jX})\times(1-e^{-j2Y}-j2Ye^{-j2Y})+(1-e^{-jX})(1-e^{-j2Y})\Big]+(1-e^{-jX}-jXe^{-jX})(1+\frac{X+2Y}{X}e^{-j2Y})+(1-e^{-jX})\Big[-j2Ye^{-j2Y}+j\frac{X+Y}{X^2}\times2(1-jX-e^{-jX})e^{-j2Y}+(1-e^{-j2Y}+e^{-jX}e^{-j2Y})\Big]+(1-e^{-jX}-j2X).\tag{9}$$

Now the oscillations can be most expected, when the negative resistances occur in both gaps G, A and G, K. On the other hand, from $\alpha_1T_1=\alpha_2T_2$, it follows that

$$\frac{T_1}{T_2}\frac{V_g}{V_g+V_a}=\frac{d_1}{d_2}=\begin{cases}\varepsilon_a\\ \varepsilon_b/(1+\varkappa)\\ -\varepsilon_c/\varkappa\end{cases}\tag{10}$$

must be fulfilled. We present these possibilities in some numerical examples following.

i) $X=\pi/2,\quad Y=\pi/2,\quad T_1/T_2=1.$

The condition that the negative resistances occur both between G and A, and between G and K is

$$0.3<\frac{V_g}{V_g+V_a}=\frac{d_1}{d_2}<1 \text{ for (a)},\quad \frac{0.3}{1+\varkappa}<\frac{V_g}{V_g+V_a}=\frac{d_1}{d_2}<\frac{1}{1+\varkappa} \text{ for (b)}.$$

These are possible for a tube with $d_2>d_1$, making $T_1=T_2$.

ii) $X=\frac{1}{4}\pi$, $Y=\frac{3}{4}\pi$, $T_1/T_2=1/3$.

The condition is

$$-\frac{0.2}{\varkappa}<\frac{1}{3}\frac{V_g}{V_g+V_a}=\frac{d_1}{d_2}<\frac{0.37}{\varkappa} \quad \text{for (c).}$$

This is possible for a tube with $d_2>3d_1$, making $\varkappa<1.11$, and $T_1/T_2=1/3$.

iii) $X=\frac{1}{2}\pi$, $Y=\frac{3}{2}\pi$, $T_1/T_2=1/3$.

The condition is

$$-0.05<\frac{1}{3}\frac{V_g}{V_g+V_a}=\frac{d_1}{d_2}<1.67 \quad \text{for (a),}$$

$$-\frac{0.05}{1+\varkappa}<\frac{1}{3}\frac{V_g}{V_g+V_a}=\frac{d_1}{d_2}<\frac{1.67}{1+\varkappa} \quad \text{for (b).}$$

These are possible for a tube with $d_2>3d_1$, making $T_1/T_2=1/3$.

iv) $X=\frac{1}{2}\pi$, $Y=\frac{5}{2}\pi$, $T_1/T_2=1/5$.

The condition is

$$-0.17<\frac{1}{5}\frac{V_g}{V_g+V_a}=\frac{d_1}{d_2}<2.6 \quad \text{for (a),}$$

$$\frac{-0.17}{1+\varkappa}<\frac{1}{5}\frac{V_g}{V_g+V_a}=\frac{d_1}{d_2}<\frac{2.6}{1+\varkappa} \quad \text{for (b).}$$

These are possible for a tube with $d_2>5d_1$, making $T_1/T_2=1/5$.

$X+Y=\pi$ means that the oscillating frequency is Barkhausen-Kurz frequency and $X+Y=n\pi$ with n (integers) means the oscillation at its dwarf waves. It has been shown that the oscillation can be expected at B.K. frequency and its dwarf wave frequencies. But the author never means to present the limit of the oscillation.

MEMOIRS OF THE COLLEGE OF SCIENCE, UNIVERSITY OF KYOTO, SERIES A,
Vol. XXV, No. 1, Article 4, 1947.

On a Generalization of the Projective Deformation

By

Joyo Kanitani

(Received December 26, 1946)

1. Let $\boldsymbol{R}_2$ be a two-dimensional space with a projective connexion $\Gamma^{\alpha}_{\beta i}\,dx^i$ ($\alpha, \beta=0, 1, 2$; $\Gamma^{\alpha}_{0i}=\delta^{\alpha}_i$). The triangle of reference $[C, C_1, C_2]$ in a projective plane $\boldsymbol{S}_2$ which moves along the development of a curve L drawn on $\boldsymbol{R}_2$ is defined by

$$(1\cdot1)\qquad \begin{cases} dC=dx^i C_i\,, \\ dC_i=\Gamma^{\alpha}_{ij}\,dx^j\,C_{\alpha} \qquad (i=1, 2\,;\ C_0\equiv C)\,. \end{cases}$$

Consider another space $\boldsymbol{R}'_2$ with a projective connexion. The spaces $\boldsymbol{R}_2$ and $\boldsymbol{R}'_2$ will be said to be projectively deformable to each orther, if we can realize a one to one correspondence between the points of these spaces in the following manner.

Let M and M' be any two corresponding points on $\boldsymbol{R}_2$ and $\boldsymbol{R}'_2$ respectively. Develop any homologous curves L and L' passing through M and M' respectively, taking a point C on $\boldsymbol{S}_2$ as the common image of M and M', and giving at this point a common initial position to the moving frames of reference which move along the developments of L and L' respectively. Take, in the vicinity of C, two homologous points P and Q on these developments. Then the *écart* $[PQ]$ is an infinitesimal of the third order at least with regard to the *écart* $[CP]$.

Choose the coordinates of the generating points of $\boldsymbol{R}_2$ and $\boldsymbol{R}'_2$ in such way that the corresponding points are determined by the same values of the coordinates x^i. Let $\Gamma'^{\alpha}_{\beta i}dx^i$ be the connexion for $\boldsymbol{R}'_2$. Then, the condition of the projective deformability of $\boldsymbol{R}'_2$ to $\boldsymbol{R}_2$ is written

$$\Gamma'^{l}_{ij}+\Gamma'^{l}_{ji}=\Gamma^{l}_{ij}+\Gamma^{l}_{ji} \qquad (l, i, j=1, 2).$$

We see therefore that if two spaces $\boldsymbol{R}_2$ and $\boldsymbol{R}'_2$ are projectively

deformable to each other, *the geodesic lines are in correspondence*, and that if two spaces with normal projective connexion are projectively deformable to each other, they are equivalent.

We can make the given connexion $\Gamma^{\beta}_{\alpha i}dx^i$ symmetry without changing the geodesic lines. We shall suppose hereafter that this transformation is previously made, namely, we have

$$\Gamma^{\alpha}_{ij}=\Gamma^{\alpha}_{ji} \qquad (\alpha=0, 1, 2;\ i, j=1, 2).$$

2. The space $\boldsymbol{R}_2$ can be plunged into a four-dimensional projective space $\boldsymbol{S}_4$ in such way that the space $\boldsymbol{R}_2$ becomes a ruled surface V_2 in $\boldsymbol{S}_4$, and that the osculating planes at a point P of this surface to various curves on the surface passing through P are not all contained in a three-dimensional projective space. We can take any ∞^1 geodesic lines of $\boldsymbol{R}_2$ as the lines becoming the generating lines of V_2. If we designate these geodesic lines, the surface V_2 depends on eight arbitrary functions of an argument[1].

Choose the coordinates x^1, x^2 so that the ∞^1 geodesic lines in question are given by $x^2=$const. Then we have $\Gamma^2_{11}=0$. Consider, in $\boldsymbol{S}_4$, a moving frame of reference $[A, A_1, A_2, A_3, A_4]$ which depends on the parameters x^1, x^2.

Let

$$dA_{\sigma}=H^{\tau}_{\sigma i}A_{\tau} \qquad (\sigma=0, 1, 2, 3, 4)$$

be the system of equations defining the movement of this frame[2]. If the vertex A describes the surface V_2 as the frame moves, we can make

$$H^{\beta}_{\alpha i}=\Gamma^{\beta}_{\alpha i} \quad (\alpha, \beta=0, 1, 2), \qquad H^{p}_{0i}=H^{p}_{1i}=0 \quad (p=3, 4),$$

$$H^{\beta}_{3i}=R^{\beta}_{112} \quad (\beta=0, 1, 2), \qquad H^{3}_{31}=\Gamma^{1}_{11}-\Gamma^{2}_{12}, \quad H^{4}_{31}=0,$$

$$H^{3}_{41}=R^{2\beta}_{12}+H^{\beta}_{32}, \quad H^{3}_{41}=\Gamma^{1}_{12}-\Gamma^{2}_{22}+H^{3}_{32}, \quad H^{4}_{41}=\Gamma^{2}_{12}+H^{4}_{32},$$

while H^{τ}_{pi} are determined by the normal system of partial differential equations:

---

1. J. Kanitani: 'Sur l'espace à connexion projective majorante, II.', *Jap. Jour. of Math.*

2. We denote by α, β, γ the surffixes which take the values 0, 1, 2; by h, i, j, k, l, m those which take the values 1, 2; by σ, τ, ρ those which take the values 0, 1, 2, 3, 4; by p, q, r those which take the values 3, 4.

$$\frac{\partial H_{p2}^{\tau}}{\partial x^1}=\frac{\partial H_{p1}^{\tau}}{\partial x^2}+H_{p1}^{\rho}H_{\rho 2}^{\tau}-H_{p2}^{\rho}H_{\rho 1}^{\tau}. \tag{2·1}$$

The tangent planes of the ruled surface V_2 along the generating line AA_1 determine the hyper-plane $\Pi=AA_1A_2A_3$. We shall call it tangent hyper-plane along AA_1 of V_2. The position of this tangent hyper-plane depends only on the parameter x^2. When this parameter varies, the hyper-plane Π is enveloped by a developable hyper-surface of which the characteristic is the plane determined by the points

$$A,\ A_1,\ A_3-H_{32}^{4}A_2.$$

The surface V_2 is therfore contained in this developable hyper-surface, and the characteristic is the tangent plane of V_2 at the point

$$E=A_1-H_{41}^{4}A.$$

We shall name it *stationary point*, for it remains unchanged when the parameter x^1 varies. The three hyper-planes Π, $d\Pi$, $d^2\Pi$ intersect at a line l passing through E, while the four hyper-planes Π, $d\Pi$, $d^2\Pi$, $d^3\Pi$ intersect at a point E_1 on l, which will be called *cuspidal point*. When the point A describes the ruled surface V_2, the cuspidal point E_1 describes a curve which will be called edge of regression, and consequently the tangent l to this curve at E_1 describes a ruled surface U_2. The osculating plane to the edge of regression at E_1 is the characteristic of the envelope of the tangent hyper-plane Π of V_2 along the generating line AA_1. This plane touches U_2 along l and touches V_2 at the stationary point E.

3. Consider a particular point A on V_2. We may suppose that $x^1=0$, $x^2=0$ for this point. The vertices A_1, A_2 of the frame $[A, A_1, A_2, A_3, A_4]$ lie on the tangent plane of V_2 at the point A. If we develop any curve drawn from a point C on V_2 into the tangent plane S_2 at C, defining the moving frame $[C, C_1, C_2]$ by means of the equations (1·1), and giving initial values so that $C_\alpha=A_\alpha(\alpha=0, 1, 2)$ for $x^1=x^2=0$, the points $C_\alpha+dC_\alpha$ are obtained by projecting the points $A_\alpha+dA_\alpha$ from the line A_3A_4. We shall name this line *axis of projection*.

Remarking that we can determine the integrals H_{p2}^{τ} of the system of partial differential equations (2·1) by giving arbitrarily the functions $\Phi_{p2}^{\tau}(x^2)$ to which H_{p2}^{τ} are reduced for $x^1=0$, we can prove the following proposition.

The space $\boldsymbol{R}_2$ can be so plunged in $\boldsymbol{S}_4$ that it becomes a ruled surface V_2 passing through a point A arbitrarily given in $\boldsymbol{S}_4$ and, at this point, the characteristic of the envelope of the tangent hyper-plane along the generating line intersects the axis of projection in $\boldsymbol{S}_4$, the point of intersection becoming the cuspidal point.

Let $\boldsymbol{R}_2'$ be a space which is projectively deformable to $\boldsymbol{R}_2$. Take the coordinates so that the corresponding points are determined by the same values of the parameters. Plunge also $\boldsymbol{R}_2'$ into $\boldsymbol{S}_4$ in the above said way, giving to the moving frames a common initial position for $x^1=x^2=0$.

Then the homologous stationary points E and E' are in coincidence for $x^1=x^2=0$, but the tangents at these points to the loci of the stationary points are different. We may choose the space $\boldsymbol{R}_2'$ so that the tangent to the locus on V_2' becomes an arbitrarily given line through E on the plane AA_1A_3.

MEMOIRS OF THE COLLEGE OF SCIENCE, UNIVERSITY OF KYOTO, SERIES A,
Vol. XXV, No. 1, Article 5, 1947.

Iteration of Elliptic Functions

By

Toshizô Matsumoto

(Received February 5, 1947)

---

1. Let $F(z)$ be an elliptic function with periods ω, ω' and its fundamental parallelogram be D. In the following we shall consider the iteration of $F(z)$. Let $W=m\omega+n\omega'$ be any period, then we consider the point ζ such as

$$F(\zeta)=\zeta+W. \tag{1}$$

The existence of such a point may easily be known provided $|W|$ is great. For let ν be the order of $F(z)$, then in D there are just ν poles. Hence we may suppose $F(z)$ bounded on the contour of D, (or else a little modification of it is sufficient). We consider the integral along the contour

$$\Im=\frac{1}{2\pi i}\oint\frac{F'(z)-1}{F(z)-z-W}dz. \tag{2}$$

$|W|$ being sufficiently great, the denominator shall not be zero at any point of the contour. If we make $|W|$ great, $|\Im|$ becomes however small. Therefore in D there are ν points ζ such as (1).

The set of points ζ for all W is countable and isolated, the poles in D being the points of accumulation.

Now about ζ we have the expansion

$$F(z)=\zeta+W+s(z-\zeta)+\cdots, \qquad s=F'(\zeta).$$

For n-th iteration we have

$$F_n(z)=\zeta+W+s^n(z-\zeta)+\cdots,$$

where

$$F_n(\zeta)=\zeta+W,$$

$$F_n'(\zeta)=F'(F_{n-1}(\zeta))F'(F_{n-2}(\zeta))\cdots F'(F(\zeta))F'(\zeta).$$

If, $|s|<1$, then since in a circle $(\mathfrak{S})$ sufficiently small, $\left|\frac{F(z)-\zeta-W}{z-\zeta}\right|$ becomes less than a number $\sigma<1$, we have

$$|F_n(z)-\zeta-W|<\sigma^n|z-\zeta|, \quad n=1, 2, \ldots.$$

ζ is (the equivalent point in D of) an attractive point and all points of the circle $(\mathfrak{S})$ by the iteration converge to ζ. All points of a certain domain (Δ) containing $(\mathfrak{S})$ in it shall have the same property. This is the immediate domain of attraction.

2. Next let us consider the attractive cycle. If

$$F(\zeta)=\zeta_1+W, \quad F(\zeta_1)=\zeta_2+W, \ldots\ldots, \quad F(\zeta_{m-1})=\zeta+W, \qquad (3)$$

where W mean only certain periods, then $\zeta, \zeta_1, \zeta_2, \cdots, \zeta_{m-1}$ are the cycle corresponding to ζ of order m. ζ shall be found from the equation

$$F_m(\zeta)=\zeta+W. \qquad (4)$$

The existence of such point may be proved quite in the same way as in the case $m=1$. For that

$$F_m(z)=F_{m-1}(F(z))=\ldots=F(F_{m-1}(z))$$

shall be bounded, z must not pass through the poles of $F(z)$. There are ν poles in D. Let α be one of them and β be such that $F(\beta)=\alpha$. There are such ν points in D. Not only this, we must also avoid such points β in D: $F(\beta)=\alpha+W$. There are ν points β in D. Now varying W, we know that for a pole, there are always countably infinite number of β. Thus for ν poles in D, there are countably infinite number of β in D. Again for each β, we must take care of points γ such that $F(\gamma)=\beta+W$. Continuing this, we have a countable set E of avoidable points in D. These points are isolated but converge to the poles in D. This is clear, since if $|W|$ be bounded, the points $\beta, \gamma, \cdots$ are finite in number, therefore, for that they are infinite in number, it must be such that $W\to\infty$, so that $F\to\infty$. Therefore we may suppose that any point of E is not on the contour of D, so that $F_n(z)$ is bounded on the contour.

Now we seclude the poles of $F(z)$ by small circles, then there

remain in D a finite number of points of E. We also seclude all the essential singularities of $F_n(z)$ by circles. Let D' be the remaining domain. There remain only a finite number of poles of $F_m(z)$ in D'. Now consider the integral as (2),

$$\Im=\frac{1}{2\pi i}\oint\frac{F_m'(z)-1}{F_m(z)-z-W}dz, \qquad (5)$$

we may conclude (4). As it is known, we have

$$F_m'(\zeta)=F'(\zeta_{m-1})F_{m-1}'(\zeta)=\cdots=F'(\zeta_{m-1})F'(\zeta_{m-2})F'(\zeta),$$
$$F_m'(\zeta)=F_m'(\zeta_1)=\cdots=F_m'(\zeta_{m-1}).$$

Hence in attractive case, for all z in a circle $(\mathfrak{S})$ about ζ, $F_{pm}(z)$ tend to ζ for $p\to\infty$; $F_{pm+1}(z)$ to ζ_1+W; $\cdots$; $F_{pm+m-1}(z)$ to $\zeta_{m-1}+W$, which we say $F_n(z)$ converges uniformly to the cycle.

3. Writing $F(\xi)=z+W$, where ζ is an attractive point such as $F(\zeta)=\zeta+W$, we consider the inverse function about $\xi=\zeta$. If ζ be not a branch point, then

$$\xi=F_{-1}(z+W)=\zeta+\bar{s}(z-\zeta)+\cdots,$$

where $s\bar{s}=1$, hence $|\bar{s}|>1$. If a circle $|z-\zeta|<r$ be in the domain of attraction, then its transformed domain by $\xi=F_{-1}(z+W)$ contains it. Therefore again we may consider the inverse function

$$\xi=F_{-1}(F_{-1}(z+W)+W),\ \cdots.$$

If these functions are holomorphic in the circle then they must be bounded in it. Hence they form a normal family. This contradicts the fact that ζ is a repulsive point of these functions. Thus about the attractive point there must be a branch point of the inverse function of $F(\xi)=z+W$, that is $F'(\zeta)=0$. Since $F'(\xi)$ is elliptic, such points are finite in number in the fundamental parallelogram; hence the number of the attractive points is finite in the parallelogram.

Same consideration will be possible for the cycle.

of extremely high electron temperature. Assuming such a high temperature of order some million degrees, it follows that the corona must emit a weak ultraviolet radiation. In the present paper, we have estimated the amount of it and tried to seek the source of excitation of the chromospheric He anomaly in it.

§ 2. *Radiation from the corona.* We assume that the chemical composition of the corona is the same as that of the reversing layer. In the coronal region, hydrogen is ionized strongly. Let N_e be the particle density of the electron. This is also equal to the density of proton. Then the energy emitted by the free-free transition per sec. per c. c. in the frequency range $(\nu, \nu+d\nu)$ is given by

$$N_e^2\phi_\nu d\nu = N_e^2 Z^2 \frac{2^7\pi^3}{(6\pi)^{3/2}}\left(\frac{m}{kT_\varepsilon}\right)^{1/2}\frac{\varepsilon^6}{c^3m^2}e^{-h\nu/kT_\varepsilon}\,d\nu, \qquad (2\cdot1)$$

where $Z\varepsilon$ means the nuclear charge. For hydrogen, we take $Z=1$. T_ε is the electron temperature, m the electron mass, c the light velocity, and k the Boltzmann constant.

Integrating it with respect to ν from zero to infinity, we find for the total energy emitted per sec. per c.c. as

$$N_e^2\Phi = N_e^2 Z^2 \frac{2^7\pi^3}{(6\pi)^{3/2}}\left(\frac{kT_\varepsilon}{m}\right)^{1/2}\frac{\varepsilon^6}{hc^3m}. \qquad (2\cdot2)$$

Inserting numerical values in the expression (2·1), we get

$$N_e^2\phi_\nu d\nu = N_e^2\, 6.67\times10^{-38}T_\varepsilon^{-1/2}e^{-h\nu/kT_\varepsilon}\,d\nu, \qquad (2\cdot1)'$$

and for (2·2),

$$N_e^2\Phi = N_e^2\, 1.40\times10^{-27}T_\varepsilon^{1/2}.$$

Compared to the free-free transition, the emission due to the bound-free transition is weaker for the electron temperature of the order of million degrees; emission per sec. per c.c. by the capture of electron to the j-th level is

$$N_e^2 Z^4 \frac{2^9\pi^5}{(6\pi)^{3/2}}\frac{\varepsilon^{10}}{m^2c^3h^2}\left(\frac{m}{kT_\varepsilon}\right)^{3/2}\frac{1}{j^3}e^{\chi_j/kT}\,e^{-h\nu/kT_\varepsilon}\,d\nu$$

$$= N_e^2 Z^4 \frac{2.10\times10^{-32}}{T_\varepsilon^{3/2}j^3}e^{\chi_j/kT_\varepsilon}\,e^{-h\nu/kT_\varepsilon}\,d\nu, \qquad (2\cdot3)$$

where χ_j means the ionization potential from the j-th level. Inte-

grating (2. 3) with respect to ν from χ_j to infinity, we find

$$N_e^2 Z^4 \frac{2^9\pi^5}{(6\pi)^{3/2}} \frac{\varepsilon^{10}}{m^{1/2}c^3h^3} \frac{1}{(kT_\varepsilon)^{1/2}} \frac{1}{j^3} = N_e^2 Z^4 \frac{4.39\times10^{-22}}{T_\varepsilon^{1/2} j^3}. \qquad (2\cdot4)$$

From the dependence of these formulae on Z, we can infer that the emissions from the metallic ions are small compared with that of hydrogen owing to their small abundance.

§ 3. Consider the concentric shell of the corona of thickness dr, whose distance from the center of the sun is r. According to (2·1), the $(\nu, \nu+d\nu)$ radiation emitted per sec. by the $(r, r+dr)$ shell is $N_e^2 \phi_\nu 4\pi r^2 dr$.

A fraction W of this, flows into the chromosphere, where W is the dilution factor of a point at distance r:

$$W=\frac{1}{2}\left\{1-\sqrt{1-\left(\frac{R}{r}\right)^2}\right\}, \qquad (3\cdot1)$$

in which R means the radius of the upper boundary of the chromosphere. Assuming the chromosphere as a layer of thickness 10,000 km, we tentatively take $R=705{,}000$ km. Thus the incident flux from the coronal region, per sec. per square cm. is given by

$$\pi F_\nu=\int_R^\infty W\left(\frac{r}{R}\right)^2 N_e^2 \phi_\nu\, dr. \qquad (3\cdot2)$$

Density distribution N_e has been derived from observations by S. Baumbach[1]. ϕ_ν is indepedent or r, if we assume a constant electron temperature. After the numerical integration we get

$$\int_R^\infty W\left(\frac{r}{R}\right)^2 N_e^2 dr=2.6\times10^{26}. \qquad (3\cdot3)$$

Therefore, we have

$$\pi F_\nu=1.75\times10^{-11} T_\varepsilon^{-1/2} e^{-h\nu/kT_\varepsilon}. \qquad (3\cdot2)'$$

By the way, we shall calculate the flux at a distant point r_1. It is given by

$$\pi F_\nu(r_1)=\left(\frac{R}{r_1}\right)^2\int_R^\infty (1-W)\left(\frac{r}{R}\right)^2 N_e^2 \phi_\nu\, dr, \qquad (3\cdot4)$$

or numerically,

1. S. Baumbach: *A. N.*, **263**, 121, (1937).

$$\pi F_\nu(r_1)=\left(\frac{R}{r_1}\right)^2 2.2\times10^{-11}T_\varepsilon^{-1/2}e^{-h\nu/kT_\varepsilon}. \qquad (3\cdot4)'$$

Coronal ultraviolet radiation will be responsible for the anomalous spectra of comets and the upper atmosphere of the earth. The above formula may be used for the consideration of them.

§ 4. *Contact temperature.* Spectral energy distribution of the free-free emission is seen from $(2\cdot1)'$ or $(3\cdot2)'$. It, of course, differs from the Planck's distribution. But it may be convenient to express the intensity at any given frequency ν in terms of the temperature radiation. From $(3\cdot2)'$ we know that the mean intensity of ν-radiation is

$$J_\nu=\frac{1}{4\pi}C\frac{1}{T_\varepsilon^{1/2}}e^{-h\nu/kT_\varepsilon}, \quad C=1.75\times10^{-11}. \qquad (4\cdot1)$$

Equating it to the Planck formula $\frac{2h\nu^3}{c^2}e^{-h\nu/kT}$, and solving for T, we find

$$\frac{1}{T}=\frac{1}{T_\varepsilon}+\frac{k}{h\nu}\ln\left(\frac{8\pi h\nu^3}{c^2}T_\varepsilon^{1/2}C^{-1}\right). \qquad (4\cdot2)$$

Let χ be the energy $h\nu$ expressed in eV. Inserting numerical values we get

$$\frac{1}{T}=\frac{1}{T_\varepsilon}+\frac{1.97\times10^{-4}}{\chi}\left(8.18+\frac{1}{2}\log T_\varepsilon+3\log\chi\right). \qquad (4\cdot2)'$$

In the following, we shall call T the contact temperature.

At the Lyman limit, $\chi=13.53\ eV$. Assuming $T_\varepsilon=10^6$, we get for the contact temperature $T=4710$. This is lower than the effective temperature of the sun. In fact, at the Lyman limit, radiation from the corona is less than one hundredth that of the Planck radiation of temperature 5700. Therefore, coronal radiation has no influence on the ionization of hydrogen. The former becomes equal to the Planck of 5700 first at $\chi=16.4\ eV$.

The above consideration shows also that the first ionizations of metallic elements are not affected by the coronal radiation, since their ionization potential lies in the range from 5 to 8 eV. The effect on the second ionization is also small. It will not change their order. For the third ionization, the coronal radiation is effective. But their concentrations are small and will be difficult to detect observationally. Thus we conclude that the coronal radia-

tion has no influence on the ionization of metallic elements. These results conserve the current interpretation on the metallic ionization and at the same time show that the high temperature hypothesis of the corona does not contradict to the observations.

He is an exception. Its ionization potential is high. The contact temperature amounts to 8000 and 16500 for the first and the second ionization potential respectively, so that the source of ionization is mainly due to the coronal radiation and a small fraction of it is in a state of ionization. But owing to its great abundance, it can amply be detected by the observation.

It is well known that the He and H is greatly enhanced in the flash spectra. In the remaining part of this paper, we shall investigate the anomaly in terms of the coronal radiation. As has been shown above, coronal ultraviolet radiation is indifferent to the hydrogen excitation. The anomalous behaviour of it may be sought for in any other sources. It may be due to the large opacity of the chromosphere to the Lyman radiation.

MEMOIRS OF THE COLLEGE OF SCIENCE, UNIVERSITY OF KYOTO, SERIES A,
Vol. XXV, No. 1, Article 7, 1947.

A New Method for Surface Temperature Measurement

By

Nobuji Sasaki

(Received March 14, 1947)

The idea of the surface temperature is now gaining importance in science, industry and medicine. The method of measurement is varied and not well standardized[1]. The surface temperature is generally affected by emission and absorption of radiation, conduction and convection due to air and evaporation and condensation of vapours. Measuring instruments now in use, such as a thermoelement or a resistance thermometer, not only interfere with these factors, but also often give false results, i. e., the temperature beneath or over the surface. Radiation pyrometry requires the knowledge of emissivity.

In the new method here proposed use is made of a thermoelement[2], whose measuring junction may be heated or cooled to any temperature. A meter inserted in the element circuit naturally indicates the temperature of the junction itself[3], but can also give the temperature of a surface, if the junction temperature has been so adjusted, that no deflection occurs on the meter when the junction is brought into contact with the surface[4], because the absence of deflection means the absence of temperature difference

---

1. F. C. Houghten and H. T. Olson: 'Measurement of Surface Temperatures', *Temperature*, American Institute of Physics, New York, (1941), pp. 855—861.

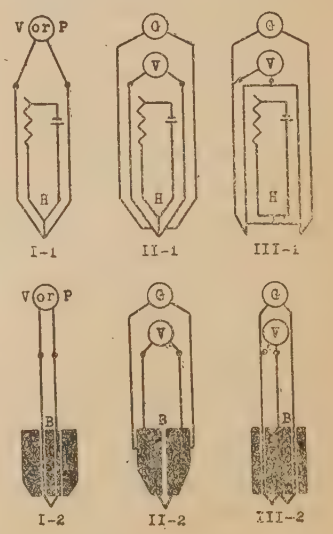
2. In principle a resistance thermometer or even a liquid-in-glass thermometer can be used as well.

3. Judged from the mode of heating, the temperature distribution inside the junction is possibly not so uniform in actual use as when calibrated in a bath with uniform temperature. Error due to this effect may, however, be very small. The surface of a silver or copper block maintained at a definite temperature may serve as a conventional surface of standard temperature, especially when used in the same inclination as the actual surface and just in front of it.

4. For a wet or a liquid surface a non-wetting junction should be used.

between them. The measuring junction may conveniently be kept constant at any temperature by metallic conduction from an electrically heated wire *H* (type 1), or given a smoothly changing temperature by varying the heating current or by means of a previously heated or cooled metal block *B*, in which the leads near the junction have been embedded with suitable electrical insulation (type 2), or else heated by radiation (type 3).

Exact results may be obtained, if the temperature jump which occurs in the junction when it touches the surface, is large for a given temperature difference between them. Such will be the case, if the surface has a high thermal conductivity and a heat capacity far exceeding that of the junction, and further if the junction has a large contact surface, a small heat capacity and a small heat loss due to conduction, convection and radiation, though some of these latter conditions are contradictory among themselves and also with the requirement for the junction to have sufficient mechanical robustness and material connection[1] with a heat source. As to the meter to be inserted in the thermoelement circuit we may use a millivoltmeter *V* (less exact) or a potentiometer *P* (more exact but awkward) (I), but exactitude and convenience can both be achieved with a millivoltmeter *V* in the full *emf* circuit and a zero instrument such as a sensitive short period galvanometer *G* either in a circuit in which the *emf* is partly compensated by the reference junctions maintained nearly at the same temperature as the measuring junction (II), or in a circuit with *emf* almost completely compensated by a symmetrically connected secondary thermoelement (III). These electrical circuits with each of the two types 1 and 2 of measuring junction are given in the figures.



To measure the surface temperature with a measuring junction type 1 we proceed thus: first, adjustment of the junction to a certain proba-

1. Such is unnecessary for the junction type 3.

ble temperature, secondly, a brief contact with the surface, thirdly, a suitable adjustment according to the magnitude and the direction of deflection; this process is repeated until no deflection occurs. The junction temperature finally adjusted is the surface temperature. To make measurement with a measuring junction type 2 it is brought to a suitable initial temperature and left to change smoothly. Brief contacts are then repeated regularly. At first the deflection will decrease and pass through zero to increase again in the opposite direction. The junction temperature corresponding to zero deflection is the surface temperature. To be precise, the temperature of junction type 1, which has been adjusted to T at some distance from the surface, will always change to $T+\Delta T$ just before the contact is made[1]. $T+\Delta T$ is then obviously equal to the surface temperature if no deflection occurs. To minimize this inconvenient ΔT the junction should be adjusted at a point as near the surface as it does not sensibly affect the surface temperature.

The validity of the new method has been roughly tested by measuring surface temperatures along a steam-fed tube with varying thickness (with conical bore), using an iron-constantan element and a millivoltmeter. The value extrapolated to zero thickness was found to be 100 ± 1°C. An automatic rapid performance of the instrument is in view.

The present investigation was carried out by the Education Ministry grant for natural science.

1. This is due to the temperature gradient in the ambient air and to the radiation from the surface, apart from irregular air draughts. ΔT depends on the speed of approach and apparently also on the quickness of response on the part of the meter.

These Memoirs are on Sale at

KÔTÔ-SYOIN COMPANY, LTD.
HUYAMATI-DÔRI, NIZYÔ-AGARU,
NAKAGYÔ-KU, KYÔTO, JAPAN

昭和二十二年三月十五日印刷
昭和二十二年三月三十日發行

編輯兼發行者　京都帝國大學理學部

印刷者　日本寫眞印刷株式會社
代表者　鈴木直樹
京都市中京區壬生花井町三

發賣所　株式會社　髙桐書院
京都市中京區麩屋町通二條上ル

CONTENTS

On the Structure of Inter-crystalline Boundaries of Metals. I
by Masaharu Kabata 1
One Possibility of Oscillation caused by Electrons in a Constant Magnetic Field
by Isao Takahashi 11
On the Possibility of Barkhausen-Kurz Oscillation. I
by Isao Takahashi 15
On a Generalization of the Projective Deformation
by Joyo Kanitani 23
Iteration of Elliptic Functions
by Toshizô Matsumoto 27
On the Ultra-violet Radiation from the Solar Corona. I
by Shôtarô Miyamoto 31
A New Method for Surface Temperature Measurement
by Nobuji Sasaki 37